ELEMENTARY IDEAS DEFINITIONS AND LAWS IN DYNAMICS

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DYNAMICS

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PREFACE.

The following pages are not intended for the use of very young students, but rather for those who, having taken such a course of physics as is now common in schools which prepare boys for college, would fit themselves to deal with the fuller developments or wider generalizations of physical science.

Such students might be referred at once to Clerk Maxwell's *Matter and Motion;* but they would find in that book some things too difficult for them, and they would hardly be led by it to make for themselves those experiments which are for many students the only means of attaining sound and vital notions of general truths. Moreover, the term inertia, which the student is sure to encounter in his reading of physics and is likely to find used in more than one sense, is ignored by Maxwell in the treatise mentioned.

The author of this pamphlet had the good fortune to find the Harvard Physical Club, at its formation last spring, willing to devote several meetings to the discussion of his manuscript. By far the greater part of this discussion was given to the pages dealing with inertia and mass, and the final *form* which those pages have taken is due very largely to criticisms and suggestions

made at these meetings. It must be understood, however, that the author takes the sole responsibility for the teachings of the pamphlet.

My thanks are due to several friends who have assisted in preparing the manuscript for the press or in reading the proof-sheets.

E. H. HALL.

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ELEMENTARY IDEAS, DEFINITIONS, AND LAWS, IN DYNAMICS.

Introductory. — Almost every beginner in the study of Physics has difficulty in mastering the ideas necessary for an intelligent handling of problems involving the consideration of masses undergoing change of motion. A large part of this difficulty arises from the fact that in such problems the student needs to use with a strict meaning terms that he habitually uses with a loose meaning, or with several meanings. The text-books do not always succeed in setting him right in this particular, and he may suffer for years under disadvantages which a few hours of study should remove. Popular significations need not be called *incorrect* by the scientific man, but frequently they are different from the scientific significations. Even scientific men do not always agree about definitions.

An attempt is here made to define and correlate the ideas associated with the words Weight, Inertia, Mass, Force, Work, and Energy. In this process any one of these terms may be used before it is defined, in cases where the popular use of the term is sufficiently exact for the immediate purpose. One word, however, Force, which must appear very frequently in the following pages, has been used so loosely, in scientific as well as in popular writings, that it is well to give at once the following provisional definition: Force is a Push or a Pull.

Weight. — It is one of the commonest facts of observation that bodies when unsupported fall toward the

earth. Whether the earth really attracts them, or whether they are by some other means impelled toward it, we do not know. The notion that the earth attracts them accounts perfectly well for the observed facts, and as no other hypothesis that has been proposed accounts for them equally well, physicists hold and teach as an extremely convenient and useful theory, if not a final fact, that bodies fall because they are pulled downward by the earth's attraction. With this explanation we shall henceforth speak of this supposed attraction as if it were an established fact.

It is another fact of observation that the earth revolves upon an axis, and it can easily be shown by experiment that the effect of this revolution must in some measure neutralize the earth's attraction, so as to make the apparent attraction slightly different from the real. The Weight of a body, in the scientific sense, is a *force*, which force is the *apparent attraction* of the earth.

Since the earth is not a perfect sphere, since it is composed of different materials in different places, and since bodies near the poles revolve in a smaller daily circle than bodies nearer the equator, we might expect the weight of a body to depend somewhat upon its place on the earth's surface. Observation shows this to be true. A given body will stretch a spring-balance more in regions far north or far south than in regions near the equator.

where is directed?

At the equator and the poles the weight of bodies is directed toward the earth's centre. At other parts of the earth's surface weight is directed nearly, but in general not exactly, toward the earth's centre. Evidently the free surface of still water or any other liquid must be everywhere at right angles to this force. Such a surface is called a *level* surface.

We ask here a question which will prove very impor-

tant when we come to speak of Inertia. When a body is placed upon a level surface in such a manner that it does not tend, from mere imperfect balancing, to roll or tip, does the weight of the body, as we have defined weight, either help or hinder the body's motion in any direction along the level surface, save by causing friction with the surface? All experience goes to show that it does not do so; that, in fact, no force in a given direction either directly helps or directly hinders motion in a line at right angles to the direction of the force. We shall therefore take it as settled, that upon a level surface the *weight* of a body affects motion only by causing friction.

Suspending a body by a fine wire we can get rid of nearly all friction except that offered by the air. When a body is so suspended and is hanging at rest, any swing imparted to it will involve some ascent by the body against the earth's pull; but if the wire used for the suspension be a long one, the curved incline up which the body moves in leaving its equilibrium position will be at first very gradual. Experience has taught every one that the pull required to move a body, with uniform velocity, along such an incline, is very small compared with that force which we call the weight of the body.

Inertia. — All this being understood, let us suspend a fifty-pound ball by a long wire, attach a string to the ball, and, taking the free end of the string in the hand, suddenly move the hand horizontally away from the ball. As soon as the string is drawn straight, one feels that the hand is *pulling*, that it encounters a *resistance*, which is offered in some way by the ball at the other end of the string. If the hand be jerked violently, this pull may become so great as to break the string, even if the latter is strong enough to bear easily the whole weight of the fifty-pound ball.

How can we account for this observed resistance? Per-

sons unaccustomed to scientific definitions would try to account for it by saying that the ball is heavy. But to say that a body is heavy means strictly only this, that the body is strongly attracted by the earth, and we have seen that such attraction cannot in this case account for any considerable resistance. The resistance of the air is evidently inadequate to explain the observed fact. In short, the closest scrutiny fails to show any cause, external to the ball, which can account for the resistance offered by the ball and felt by the hand. We are thus led to the conclusion that the source of this resistance lies in the ball itself. This does not explain the resistance. It merely *locates* it, which is worth doing.

In this experiment a *force* applied to the ball set it in motion from rest, overcoming a certain *resistance*. Investigation leads us to conclude that no body ever starts from rest in any other way. If a body be already in motion and we wish to make it move faster, the case is essentially the same; a force must be applied because a resistance is to be overcome.

If we provide ourselves with a heavy wheel, like a grindstone, mounted to revolve upon an axis, and a cannon-ball to roll along a level floor, we can try experiments in *destroying* motion or changing its *direction*. From all these experiments, if they be carefully made, we may draw this conclusion, which contains, in substance, Newton's First Law of Motion: *The application of a force is necessary for any change in the direction or magnitude of a body's motion, and this application of force is made necessary, not by any known external obstacle to such change of motion, but apparently by the very nature of matter.* Setting a body in motion is to be regarded as a special case of changing its motion.

In referring to the fact that bodies in certain circumstances act in certain ways, it is customary to speak of

them as possessing certain properties. Thus, in describing the behavior of steel wire we do not have to say that it can offer great resistance to a force tending to pull it apart. We express the same fact by saying that it has great *tenacity*, for common usage declares tenacity to be that property of a body which keeps it from parting without the application of force.

Following analogies like this, we for the sake of convenience and brevity speak of that behavior of bodies which we have just been discussing, as due to a certain *property* of matter. This property is called Inertia. We say, then, *Inertia is that property of matter which makes the application of a force necessary for any change in the magnitude or direction of a body's motion.*[1]

In experiments like those already suggested it will at once be observed that the force, or resistance, exerted by a body varies greatly with the conditions of the experiment, being sometimes large and sometimes small, according to the following general law: When the ball's motion is changed slowly, it offers a slight resistance, — a small force suffices; when a considerable change is to be effected in a short time, we encounter a large resistance, — a great force is required.

We must note here an important contrast between the act of setting a body in motion and the act of lifting a body. To lift a given weight requires a certain force; it cannot be lifted by a smaller force. In setting a free body in motion there is no such limitation. Continued experiment with the suspended ball will prove, or furnish strong evidence tending to prove, that there is no definite magnitude, however small, which the applied force must attain before it can set the ball in motion. Hence we believe that a body free to move cannot remain at rest under the slightest force tending to set it in motion, al-

[1] See Appendix.

though its rate of acquiring motion may be very small, if the applied force be very small. All that is here said of *setting a body in motion* is equally true of *changing its motion* in any respect. If a great mass be in motion, its motion can be slowly changed or destroyed by the smallest force, properly applied.

We have spoken sometimes of the force which *is applied* to a body to change its motion and sometimes of the resistance, or counter-force, with which the body meets the applied force. Each is necessary to the other. We could not exert force upon a body if the body offered no resistance. On the other hand, resistance would be impossible if there were no applied force to be met. We shall call the counter-force, which a body in virtue of its inertia exerts to meet a force applied, the *inertia-force.*

Inertia and *inertia-force* must be carefully distinguished, as any property must be distinguished from the exertion of the same property, as bodily strength, for instance, must be distinguished from the exercise of that strength. We do not think that a man's strength has departed merely because, for the moment, he may not be exhibiting it. We do not think of a telegraph wire as losing its electric conductivity when, the battery having been detached, the wire ceases to conduct. We consider a body as having *inertia* at all times, but its *inertia-force* exists only during change of motion, and may have any magnitude either great or small, according to the rate of change of motion.

Usually, when a force is applied to a body, the resistance by which it is met is due, in part at least, to some other applied force. Thus if the given force tends to raise the body, gravity opposes it; if the body rests upon a rough surface, friction will oppose its motion along the surface. When in such cases the opposing applied forces are equal, they neutralize each other, and there is no

change of motion, and hence no inertia-force is developed. If one of the opposing applied forces is greater than the other, the greater will prevail and a change of motion will occur, occasioning an inertia-force, which will work *with* the smaller applied force *against* the greater.

Let us now, beside our suspended fifty-pound ball, suspend in the same way by another wire a much smaller ball of the same material. Experiment will at once show that to give the same velocity in the same time to both balls a larger force must be applied to the large ball than to the small one.

Mass. — This fact suggests a method of comparing what is called the *quantity of matter* in one body with that in another body. But before setting forth the method in detail, we must consider what is meant by the phrase *quantity of matter.*

We have knowledge of matter only through its properties. We can compare bodies only in respect to these properties. When we are asked to determine whether a certain body of one material contains more or less matter than a certain body of another material, the problem is in its nature not unlike that of deciding which of two unlike men is to be considered the greater. If this latter task were before us, we should find it necessary to select some one or more particulars in which the men acted alike, or produced similar results. We might thus be able to decide that in this or that respect one man was greater than the other, but it is not unlikely that he who was found to be greater in some particulars would prove to be less in others. In coming to a conclusion two courses would be open to us, to frame an opinion with reference to all particulars that, on the whole, the one man was greater than the other, or to select some one important particular, disregarding all others, and, trying each man by this, declare him to be greater or less according to the issue of this single test.

The first course would give us a loose judgment; the second, a strict judgment upon a limited and arbitrary basis. This latter judgment would be false, if announced, or habitually used, without recognition of its limitations, but might be highly advantageous, if its conventional character were borne in mind. For loose *on-the-whole* judgments, science has comparatively little use. So when scientific men say, for instance, that a certain piece of gold and a certain piece of silver contain, or consist of, equal quantities of matter, they mean that both bodies, whatever may be their inequality in other respects, are exactly alike in regard to some one highly essential particular.

There are several characteristics belonging, so far as we know, to all kinds of matter, which force themselves habitually upon our attention and are of such a nature that we turn to them, when we attempt to estimate the quantity of any aggregation of matter.

These characteristics are: —

Matter occupies space;

Matter attracts matter;

Matter requires the application of force to change its motion.

These facts are here put in this order, because this is the chronological order in which they are first recognized by every student.

If we were to compare two bodies of different material by the three tests thus suggested, we should find, in general, that, if the bodies had equal volumes, they would not weigh alike and would not require equal forces to produce equal changes of motion in equal times. We shall call these three tests the *volume-test*, the *weight-test*, and the *inertia-test*, respectively. Since they do not all agree, which one shall be selected and agreed upon as the best?

To a child the volume-test is the natural one, perhaps the only one he can apply or imagine. As he grows older, he observes that bodies may change in volume without addition or subtraction of substance, and that in such cases the weight remains constant. He therefore comes to prefer the weight-test to the volume-test. Continuing, he learns that a given body does not weigh the same at all parts of the earth's surface, and that in regions of space far from the earth, where nevertheless science has to deal with matter, the aspect of weight, if we regard it at all, is quite changed. He finds, however, that there is every reason to believe that a given body, in whatever part of space it might be placed, would require the same force to give it the same velocity in the same time. He therefore in the end comes to regard the inertia-test as more widely applicable than the weight-test. He now agrees with other physicists that two bodies which are equal in the inertia-test shall be said to contain, or consist of, equal quantities of matter.

As an equivalent for the phrase *quantity of matter*, the word Mass is commonly used. *Equal Masses*, then, are, *by definition*, quantities of matter which, *whatever their inequality in other respects*, are alike in this, *that they require equal forces to give them equal velocities in equal times.*[1]

In spite of this fact, that by definition the inertia-test is the basis for the comparison of masses, they are in practice very seldom compared by direct application of this test. The reason for this is that the inertia-test is difficult to apply. Moreover, whenever it is applied with great care and accuracy, it is found that bodies which are equal in this test are equal in the weight-test also, both bodies being weighed at the same place. Hence, if our task is simply to compare the masses of two bodies which can be brought to the same part of the earth's

[1] See Appendix.

surface, we get with very little difficulty by weighing the same result of comparison that we should get with great difficulty by applying the inertia-test directly.

This agreement of the weight-test and the inertia-test is of great practical convenience to us in general, but it has its inconvenient side, for in comparing masses habitually by weighing there is danger of overlooking the importance of inertia, and this neglect leads to very absurd conclusions, when one undertakes to imagine the behavior of bodies emancipated wholly or in part from the influence of gravitation.

For the very reason that the inertia-test is seldom employed, it may be well to describe with some detail a simple and rude example of its use. To avoid confusion of ideas it will be well to make this process free, as far as possible, of all reference to gravitation. We will, therefore, following the suggestion given in Maxwell's "Matter and Motion," take elastic strings as the means for applying the necessary forces. Thin strips of india-rubber found in toy-stores serve well for this purpose. Take two such strips equal in length and as like as may be in width and thickness. Put these strips in line, the end of one slightly overlapping the end of the other. Clamp these joining ends firmly together and then stretch the whole line until it is two or three times as long as when unstretched, and fasten it in this condition. Probably one strip will have stretched less than the other. If so, trim its edges until the clamp which holds the joining ends of the strips is drawn to the middle point of the whole line. The strips may now be separated. Hereafter they will, until their properties suffer some change, exert equal pulls when equally stretched. Provide now a pair of roller-skates, of the simplest pattern [1] but well

[1] Such skates have been for sale recently at "junk" and hardware stores in Boston for twenty-five or fifty cents a pair.

oiled so as to run as freely as possible. Place these side by side on a level surface and to each skate attach one end of one of the rubber strips, making the strips parallel to each other and to the running direction of the skates. Fasten the free ends of the strips to some fixed object in such a way that the strips, when stretched by moving the skates, may be parallel to the surface upon which the skates rest. Upon each skate place a chalk-box containing sand.

If it were not for the disturbing action of friction, we could now apply the inertia-test with the greatest ease and decide at once, with considerable accuracy, which box, with its contents, has the greater mass. It would be necessary merely to draw the skates along side by side, so as to stretch the attached strips equally and to a considerable extent, then release them at the same time and observe which one was drawn back the more slowly. The load upon this skate, we should reason, would require a greater force to make it keep pace with the other, hence it must have the greater mass. (This reasoning of course assumes, as we shall assume throughout, that the skates themselves are of equal mass.) By transferring sand from one box to the other we could make the two loads equal in mass, which condition of things would be indicated by the skates keeping together in their motion.[1]

But friction complicates the case. To meet this difficulty make each skate to run on a straight, smooth board, so inclined that the skate with its load will, once started, the rubber strip being detached, roll with uniform velocity down the incline. If one skate naturally runs less freely than the other, this skate will require the steeper incline. The whole influence of friction is now neutralized by the slight pull of gravity along the inclines and the elastic strips have to deal with the inertia-forces

[1] Compare with Lodge's *Elementary Mechanics*, p. 43.

alone, so that we may proceed with the test as if we had frictionless skates rolling upon a level surface.

In this test we start with forces known to be equal, and by means of these equal forces obtain equal masses. If we had started with masses known to be equal, we could by a very similar experiment have obtained equal forces.

Units. — We can hardly go farther without the notion of units. All so-called measurement consists in determining that the quantity measured is a certain number of times as great (this number may be integral or fractional) as another quantity of the same kind, which quantity is called the unit quantity. The unit used is not always the same in measuring quantities of the same kind. Thus we may say that a certain line is ten feet long, that another is six inches long; that one body contains one hundred pounds of matter, that another body contains ten ounces of matter. In these examples the foot and the inch are both used as units of length, and the pound and ounce as units of mass. Similarly, we have various units of force, velocity, energy, etc.

It would be possible to make for each kind of physical quantity a set of units without reference to those chosen for any other physical quantity, but since these various quantities are closely related, and since we are obliged continually to take account of their relations in physical problems, convenience requires that we take account of these relations in choosing our several kinds of units.

It is well known to physicists that we can, after selecting three units of different kinds, which units may be entirely independent of each other, base upon these three all other units needed in the measurement of purely physical quantities. This will become plainer as we proceed. The three units usually selected as simple, or

fundamental, units are units of mass, length, and time; for instance, the pound, foot, and second, or the gramme, centimetre, and second. We shall not just here name specific units, but shall speak in general terms of the, or a, unit of mass; the, or a, unit of length; the, or a, unit of time; and, in equally general terms, of other units based upon these.

Starting with our fundamental units of mass, length, and time, we will define first, as one of the simplest *derived* units, that of velocity, which is the velocity that, maintained uniform, will carry the body possessing it a unit of distance in a unit of time.

We will consider next the definition of a unit of force. If we follow the general practice of English-speaking people, we shall take as the unit of force *the pull of gravity upon a unit of mass.* This definition makes no use of the fundamental units of length and time. It is based on the unit of mass and the earth's attraction. Another kind of force-unit, for which the preceding pages have prepared us, is defined as follows: *That force which, acting for a unit-time upon a unit-mass, will give it unit-velocity.* Since unit-velocity is defined by means of unit-distance and unit-time, this second definition of a force-unit is based solely upon the units of mass, length (distance), and time. The idea of unit-time is really introduced twice therein, once explicitly, and once by implication in the idea of unit-velocity.

The two definitions give quite different kinds of force-units. From the first we get what is called, for an obvious reason, the *gravitation* unit of force. We have already noted, in speaking of Weight, that the pull which the earth exerts upon a given body is different at different parts of the earth's surface, and of course it would diminish indefinitely if the body were moved farther and farther from that surface. Hence the gravitation unit

of force, as defined above, is variable, increasing and diminishing in turn as we go from place to place. The unit given by the second definition is not subject to such changes. It is called, therefore, the *absolute* unit of force. It is called also the *kinetic* unit (from κινέω), because it involves the idea of setting matter in motion.

It must not be supposed that the gravitation unit is used only in cases where gravity is involved, or that the absolute, or kinetic, unit is used only in cases involving change of motion. Either unit may be used in measuring physical forces under all conditions. The gravitation unit, being much simpler in definition than the absolute, or kinetic, unit, is the older and more familiar. In spite of its local variations, it is accurate enough for purposes of ordinary business, and when greater accuracy is required, the gravitation unit may be fitted for the demand by defining it as the pull of gravity upon the unit mass at some designated spot upon the earth's surface, London, for instance. The absolute unit is, however, so much more convenient to use in cases involving change of motion, and such cases are so very numerous in the problems of science, that this unit is generally used by physicists.

We will now consider certain specific units of mass, length, time, and force. The English-speaking world uses generally as units of mass, the pound, ounce, grain, etc., the magnitudes of which bear to each other fixed and simple ratios. Of these the pound is considered the basis, and the English government keeps a certain piece of platinum which is by Act of Parliament declared to be the legal standard of *mass* (*weight*, in the language of the Act).

English units of length in common use are the yard, foot, inch, etc., which, like the units of mass just named, are in magnitude simply related to each other. Of these,

the English government regards the yard as the basis, and it preserves a certain bar as a standard yard. Physicists more frequently use the foot as the unit of length.

Common units of time among English-speaking people are the year, day, hour, and second. For accurate scientific purposes it is customary to take as the unit of time the *second* of "*mean solar time*."[1]

For the English system, then, we have as the fundamental units, the *pound*, the *foot*, and the *second*. The corresponding unit of *velocity* is that of a body moving at the rate of one foot in one second. The *gravitation* unit of *force* is the pull of gravity upon a mass of one pound. This *force* also is called a *pound*. We have thus a *pound-mass* and a *pound-force*, each of which is frequently called a *pound*. A similar ambiguity attaches to the names of other mass-units, and should be carefully noted.

The *absolute* unit of force in the English system is *that force which, acting upon the pound-mass for one second, will give it a velocity of one foot per second.* In comparatively recent years the word *Poundal* has attained considerable currency as the name of this absolute unit of force. It previously had no one-word name. The ratio of the *pound-force* to the *poundal* will be discussed a few pages farther on.

There is another system of units which has been adopted very generally by physicists. Its fundamental units of mass, length, and time are the gramme, centimetre, and second. The whole system of fundamental and derived units is called the Centimetre-Gramme-Second, or C. G. S. system.[2] The gramme and centimetre of this system were chosen without reference to the English pound and foot, and there is no simple and exact

[1] For a fuller account of the English units of mass, length, and time, see Maxwell's *Theory of Heat*, chap. iv.

[2] See Maxwell's *Theory of Heat*, chap. iv., and the Appendix to Everett's *Units and Physical Constants*.

ratio to connect the mass-units or the length-units of the two systems. The time-unit is the same in both systems. The fundamental units of the C. G. S. system were adopted from the French, but the full development of the system is due to the British Association for the Advancement of Science.

The *gravitation* unit of force corresponding to this system is the pull of gravity upon a mass of one gramme. The absolute unit of force in the system is *that force which acting on the gramme-mass for one second will give it a velocity of one centimetre per second.* It is called the *Dyne.*

Force. — Having now obtained a proper notion of units, which enables us to speak intelligently and definitely of magnitudes, we are prepared to state more fully the law which governs the effects of forces acting to change the motion of masses. In this statement we shall, except where the contrary is expressly declared, consider forces as measured in *absolute* units. We shall, moreover, assume each force while acting to be constant in magnitude and direction. The statement may be put in the form of questions, with the answers that experiment yields: What velocity would unit-force impart to m units of mass in one second? *Ans.* $v = \frac{1}{m}$.

What velocity would unit force impart to m units of mass in t seconds? *Ans.* $v = \frac{t}{m}$.

What velocity would f units of force impart to unit-mass in one second? *Ans.* $v = f$.

What velocity would f units of force impart to m units of mass in one second? *Ans.* $v = \frac{f}{m}$.

What velocity would f units of force impart to m units of mass in t seconds? *Ans.* $v = \frac{ft}{m}$.

Since f, t, and m may be any numbers whatever, the

last equation implies the others, and is the general law connecting velocity imparted, force, time, and mass, under the conditions stated above. Any three of these four quantities being given, the fourth is at once obtained by means of this equation.

Certain forms which this equation may take are of special interest. Let us, for instance, write it thus: $f = \frac{mv}{t}$. Now, the product, mv, is called the *momentum* of the mass m, moving with the velocity v. This latter form of the equation may, therefore, be thus interpreted: *A force is measured by the momentum it can impart to any body whatever, divided by the time occupied in imparting it;* or more briefly, *a force is measured by the momentum it can impart in one second.* Put in the form $mv = ft$, the equation means this: *the momentum imparted to any mass by any force in any length of time is equal to the product of the force and the time.* This statement, taken with the fact that the velocity imparted by a force is in the same direction as the force, is equivalent to Newton's Second Law of Motion, as stated and interpreted in Maxwell's "Matter and Motion."

The formulas given above may at first appear to be applicable only to the case of a force acting alone, and setting a body in motion from a state of rest. But even if the body be moving, in any direction whatever, before the force begins to act, or if several forces act, in any directions whatever, simultaneously upon the body, all that is said above concerning the action of a single force is still applicable in this sense, that the resultant momentum, after any lapse of time, is to be found by calculating the effect which any and every force engaged would have produced working alone, and then combining the results according to the rules for the composition of velocities or forces, the original momentum entering as one of the components.

These formulas hold for the English and the C. G. S. systems of units equally well, but of course they would not hold for a mixture of the two systems, such, for instance, as reckoning force in dynes and mass in pounds. Nor do they hold, in the forms given, when the force is measured in gravitation units. We can now easily discuss the ratio of the gravitation and absolute units of force corresponding to any given unit of mass; for instance, the pound-force and the poundal. The pound-force, that is, the earth's pull upon a pound-mass, acting upon this pound-mass for one second, gives it a velocity of about 32 feet per second. Hence, *the pound-force is about 32 times as great as the poundal.* In fact, it is evident that the ratio of any gravitation unit of force to the corresponding absolute unit of force is given by the so-called *acceleration of gravity*, which is usually denoted by the symbol g. Hence, if a force employed to set a mass in motion be expressed in gravitation units, the general formula for the velocity imparted will be $v = g\,\frac{ft}{m}$. In the C. G. S. system the value of g is about 981.

We have now spoken repeatedly of a force applied to a body, but it must be observed that every force implies an action between *two* bodies, or two parts of the same body, and that this action produces an effect upon each of them. The following quotations touching this subject are passages from Maxwell's "Matter and Motion": —

"The mutual action between two portions of matter receives different names according to the aspect under which it is studied, and this aspect depends on the extent of the material system which forms the subject of our attention.

"If we take into account the whole phenomenon of the action between two portions of matter, we call it Stress. This stress, according to the mode in which it acts, may

be described as Attraction, Repulsion, Tension, Pressure," etc.

"But if . . . we confine our attention to one of the portions of matter, we see, as it were, only one side of the transaction — namely, that which affects the portion of matter under our consideration — and we call this aspect of the phenomenon, with reference to its effect, an External Force acting on that portion of matter, and with reference to its cause we call [it] the Action of the other portion of matter. The opposite aspect of the stress is called the Reaction on the other portion of matter."

These passages are quoted here mainly for the definitions they give. The facts which they state will probably be recognized as such by all who read them. We proceed to compare the magnitudes of the effects produced upon two bodies by their mutual action.

Much can be learned in this connection by experiments upon the collision of suspended balls, elastic and inelastic, swinging in arcs of equal radii and placed so as barely to touch each other when hanging at rest. There is, however, a certain advantage in using some arrangement that permits the bodies experimented upon to start, under the influence of their mutual action, from evident rest. We shall describe a rude contrivance that does this, and which illustrates the law of action and reaction with sufficient approach to accuracy to make it instructive.

Take two pint tin pails, with covers, and remove the handles. Into each of the holes from which the handles have been removed fasten a string. Suspend each pail by the two strings thus attached, making the suspension as long as practicable, and having the two strings diverge so as to be one or two feet apart at the top. Let the two pails, so arranged as to swing in the same vertical plane, hang at the same height and nearly touching each other.

Provide now a rather stiff piece of clock-spring, about

ten inches long and half an inch wide, and fasten one end of this to the outer side of one of the pails in such a way that, when the spring is bent over in a smooth arch, the other end will come between the pails and reach a little below their middles. Fasten this end in place there by means of a thread in such a way that it can be at once released by burning off the thread.

Fill the pails with sand, put on the covers to prevent spilling, and burn the thread. If the masses are equal, the pails will by the action of the released spring, which remains fastened to one of the pails and is to be considered a part of it, be sent off in opposite directions with equal velocities, and each may, if the suspensions are ten feet long, swing a foot and a half from its position of rest.

If the sand in one pail be replaced by something heavier, and that in the other by something lighter, so as to make the mass of one pail and its contents n times as great as that of the other pail and its contents, and the experiment be now repeated, the lighter pail will swing about n times as far as the heavier. Swinging n times as far along the arc of a circle, it will *rise* n^2 times as far, which implies an original velocity n times as great as that of the other pail.

In this experiment, then, however the masses may differ, the *momentum* received by either of the bodies from their mutual action is equal to that received by the other and opposite in direction. A like result will be obtained in any experiment whatever which shows the mutual action of bodies. We are thus led to Newton's Third Law of Motion, which is stated by Maxwell as follows: "*Reaction is always equal and opposite to action, that is to say, the actions of two bodies upon each other are always equal and in opposite directions.*"

The law, *reaction is always equal to action*, is readily accepted in its abstract form by young students, but

many of its applications they are inclined to reject. Imagine two men, *A* and *B*, pushing against each other, and so balanced that neither is gaining any advantage. It is easy to see that in this case *A* pushes against *B* just as hard as *B* pushes against *A*. But let *A* be gaining an advantage, so that *B* is pushed backward: is *B* now pushing against *A* as hard as *A* is pushing against *B*? Perhaps nine persons out of ten would answer no, although in so doing they would virtually deny the truth of the law, reaction is always equal to action.

The difficulty here should not be great for those who have read the preceding pages with care. The case is this: *A* is trying to push himself forward, and he encounters an opposing force at the surface of contact with *B*. *B* is trying to push himself forward, and he encounters an opposing force at the surface of contact with *A*. Will *A* go forward, or will he not? Will *B* go forward, or will he not? *A* will go forward, if the push which he gives himself is greater than the opposing force he meets at the surface of contact with *B*. *B* will go backward, if the push he gives himself is less than the opposing force he meets at the surface of contact with *A*. Now these conditions, which insure that *A* shall prevail over *B*, are evidently quite reconcilable with the further condition, which the law, *reaction is equal to action*, requires, that the push of *A against B* shall be neither greater nor less than the push of *B against A*.

Shall we say, then, without qualification, that *A* pushes no harder than *B*? No. We must observe the full significance of the phrases, *against B*, *against A*. The force which *A* exerts from behind to push himself forward, the *vis a tergo*, may be greater than the corresponding force which *B* exerts. If so, the excess of *A*'s *vis a tergo* over *B*'s is spent in overcoming the *inertia-force* incident to setting in motion the bodies of the two men.

At the beginning we defined a force, provisionally, as a *push* or a *pull;* and in no case in these pages has the word force been used in such a sense that it cannot be replaced by one or the other of the words push and pull, provided these terms are interpreted with sufficient liberality to make them cover attractions and repulsions existing without visible means of communication. It may, of course, be objected that the words push and pull are as difficult to define formally as the word force, but their merit is that they can be taken at once without being defined, since their habitual use in non-scientific speech is far more exact than that of the word force.

As a type of the more common definitions of force the following will serve: "Force is whatever changes or tends to change the motion of a body by altering either its direction or its magnitude." This definition is a good one for a person who already knows what a force is and what it is not, but it probably never conveys at once the proper notion to a young student. Let us apply it to the case of a horse starting a cart. What is it that here 'changes the motion' of the cart? The natural answer is, the horse. Is the horse, then, a *force?* Not in the scientific sense, and not according to the *intention* of the definition quoted. This definition, with its ambiguous phrase "whatever changes," is not, then, a good one for elementary teaching.

Such phrases as the *forces of nature*, which are useful and almost indispensable, use force in a different sense, but these are scarcely to be considered scientific expressions. In connection with the subject of Energy will be pointed out one of the evil results of a failure to perceive clearly the different meanings of the word force.

Work. — In popular language a man may be said to do *work* in merely *sustaining* a load. Science reserves the word work for cases in which some *change* is wrought,

and does not credit the performance of work to a man so occupied. His condition may cause certain operations in his body that will occasion fatigue, and that may, in the scientific sense, be called work; but, so long as he is not changing the condition or position of his load, he is not, according to the terms of science, *doing work upon that load.*

Maxwell says, in his "Theory of Heat," "*Work is done when resistance is overcome.*" Students sometimes misunderstand the word *overcome* in this connection, saying that we overcome the force of gravity when we merely prevent a body from falling. To *overcome resistance*, in the sense of the definition given, is to *effect or assist motion against resistance.* The body which *overcomes* the resistance is said *to do work upon* the body which *offers* the resistance. In certain cases, if not in all, either of the two bodies, according to our point of view, may be regarded as doing the work. This matter will be briefly discussed in connection with Energy.

The resistance overcome may be any one of a wide variety. It may be due to gravity, to friction, to electrical or magnetic action, to elasticity, etc. It may be due to inertia, as we have seen; work is required to set a body in motion; if this were not so, the body could do no work in losing its motion.

The unit quantity of work is defined, in general terms, as *the work done by the unit force overcoming the corresponding resistance through a unit distance.*

If we select some particular unit of force, — for instance, the poundal, which is an "absolute" unit, — and take the foot for our particular unit of distance, and repeat the above definition with these units, we get the definition of a *particular* unit of work, which may be called the *Foot-Poundal*, an "*absolute*" unit of work. If we select as our particular unit of force the pound, a

"gravitation" unit, and for our unit of distance the foot, we obtain the definition of a different particular unit of work, the *Foot-Pound*, a "*gravitation*" unit of work.

Obviously, we may so select our units of force and distance as to get absolute units and gravitation units of work in other systems. Thus we get the *Erg*, which is the amount of work done by a *dyne* acting a *centimetre*, and the *Kilogramme-Metre*, which is the work required to *lift one kilogramme* a distance of *one metre*.

From the definitions given it is evident that in point of magnitude the gravitation unit of work bears to the absolute unit of work the same relation that the gravitation unit of force bears to the absolute unit of force.

It is further evident, from the definition of the unit quantity of work, that f units of force acting through a unit distance do f units of work, that the unit-force acting through a distance of d units does d units of work, and that f units of force acting through d units of distance do fd units of work. If, then, we look at the performance of work from the point of view of the force which accomplishes it, we say that *the quantity of work done is measured by the product of the working force and the distance through which it overcomes the resisting force.*

If, on the other hand, we regard it from the point of view of the resistance overcome, we say, "*the quantity of work done is measured by the product of the resisting force and the distance through which that force is overcome.*" (Maxwell.) These two measurements give the same numerical result; for the *distance* is the same in both, and, as we have seen in discussing the law, *reaction is equal to action*, the *working* force and the *resisting* force must also be equal.

It must be noted that what is here called the *working force* is not always the same as the whole force applied. Imagine a man pushing a car by means of a horizontal

pressure not quite parallel to the track. To find the *working*, or *effective*, force in this case we resolve the total pressure which the man exerts against the car into two components, one parallel to the track and the other at right angles to the track. The first of these components is the *working*, or *effective* force. The second neither helps nor hinders the movement of the car, save incidentally by affecting the friction which is overcome by the other component.

Energy. — This is perhaps best defined as *the power of doing work.*

When a body is in motion we recognize in it a power of overcoming resistance *in virtue of that motion and its own inertia.* This kind of energy we call *Kinetic* energy.

On the other hand, we may find in a system composed of different bodies, or different parts of the same body, a state of *stress* (see definition of Stress under Force), an effort of the system to change its shape, state, or size, and, *in virtue of this stress*, a power of overcoming resistance. Such power is possessed by a system made up of the earth and any body raised above its surface, by a foot-ball distorted from its natural shape, or by a mixture of oxygen and hydrogen, etc. This kind of energy is called *Potential* energy.

Evidently, the two kinds of energy may coexist in a body. Frequently students entering college define kinetic energy as the *energy of a body in motion*, as if all the energy of a moving body must be kinetic. Some, also, will say that kinetic energy is energy which *is doing* work, thinking, for instance, that the *potential* energy of a raised pendulum is instantly turned into *kinetic* energy when the fall begins, and that whatever work is then accomplished is done by the kinetic energy.

The *amount* of any given store of energy is measured

by the amount of work it can do. *Energy* is therefore measured in the same units as *work*. There are *gravitation* units of energy, like the foot-pound and kilogrammetre, and *absolute* units of energy, like the foot-poundal and the erg.

The amount of kinetic energy possessed by a body is easily calculated, if we know its mass m and its velocity v. The momentum of the body is, by definition, mv. Let this body encounter an opposing force of f absolute units, constant in magnitude and direction. From what has been given under Force we know that the mass will under these conditions lose its momentum at the rate of f units per second. The number of seconds it will continue in motion is, therefore, $\frac{mv}{f}$, and, since its motion is destroyed uniformly, its average velocity during this time is $\frac{v}{2}$. Hence the total distance it travels is $\frac{mv}{f} \times \frac{v}{2} = \frac{mv^2}{2f}$, and the work it does against the resistance f is $f \times \frac{mv^2}{2f} = \frac{1}{2} mv^2$, *which expression, therefore, represents, in absolute units, the amount of kinetic energy the body possessed when moving with the velocity* v. If we should calculate in a similar way the work required to *impart* the velocity v to the mass m, we should of course obtain the same result.

Expressed in gravitation units, which, as we have seen, are g times as great as the corresponding absolute units, the kinetic energy of the body is evidently $\frac{mv^2}{2g}$.

The conditions upon which the potential energy of a system may depend are so numerous and so varied, that it is hardly worth while to give here formulas for the amount of such energy in any case except that of a body raised a short distance above the surface of the earth.

In dealing with such a case we assume that the potential energy of the system formed by the earth and the body in question will be entirely spent when the body reaches the level of the earth's surface. With this understanding the amount of potential energy of the system is evidently the product of the body's weight and its distance above the earth's surface. The body's weight expressed in gravitation units of force is m, and this multiplied by h, the height, gives for the potential energy mh gravitation units. The weight expressed in absolute units is mg, and the potential energy in absolute units is, therefore, mgh.

We may say in general terms, that *the amount of potential energy in a system is equal to the amount of work of all kinds that the system can do, in virtue of the state of stress existing between its parts, while changing from its present condition to that which may be considered as its natural, or standard, condition.*

What shall be considered the natural, or standard, condition of any given body, and, therefore, what amount of potential energy shall be represented to our minds by a given state of that body, depends upon the point of view from which we regard it. Thus, in one aspect the position in which a pendulum has no potential energy is that in which it hangs perpendicularly from its support. In another aspect its position of no potential energy is at the surface of the ground.

There is a somewhat similar opportunity for different views concerning the amount of kinetic energy to be attributed to any body. For instance, a stone resting in the ground is perhaps usually considered as having no kinetic energy, but if the view be broadened somewhat, so as to recognize the motion of the earth, the stone has to be considered as possessing kinetic energy in virtue of its motion with the earth.

This ambiguity which we perceive in speaking of amounts of energy affects also the consideration of work. It may be said in general, that when one body gains energy at the expense of another, the latter does work upon the former. But cases like the following have been imagined: An arrow is shot from the rear of a moving train in such a way as to destroy entirely the motion which the arrow had in common with the train, so that it will fall vertically to the ground. Does the bow do work upon the arrow, or does the arrow do work upon the bow and the train?

This is much like asking whether the arrow leaves the bow or the bow leaves the arrow. For different purposes each view in turn would properly be taken.

Conservation of Energy. — We recognize only two kinds of energy, — kinetic and potential; but special names are given to various forms in which these two kinds of energy appear, either singly or combined. Such forms are, Heat, Chemical Energy, Electrical Energy, Magnetic Energy, and "Radiant" Energy (the energy sent out in radiations from light-giving or heat-giving bodies).

All such forms are so related that energy in any one of them may be turned into any or all of the others in succession and back finally to its first shape.

In actually performing such a circle, or *cycle*, of transformations we observe an apparent loss. We bring back to the original form a smaller amount of energy than we started with. Thus a ball thrown upward returns to the hand with diminished velocity. We must not, however, conclude from such trials that there is any real destruction of energy in the processes of transformation.

If we were to take a quantity of water filling a certain vessel and pour it into a dozen vessels in succession till we reached the first vessel again, this vessel would not

now be filled, but we should not infer from this any real destruction of the substance of the water. We should account for the apparent loss by the amounts left clinging to the sides of the vessels, or absorbed by them, or spilled, or evaporated, or possibly changed by chemical action into some different form of matter; and we should have not the slightest doubt that, if all these small amounts could be collected, they would just fill the space now empty in the vessel. This confidence is the result of long experience, which has taught us to believe that matter cannot be destroyed.

Equally long experience has been teaching the somewhat more subtle truth that energy cannot be destroyed, that its apparent annihilation in its transformations, or wasting away without transformation, is to be explained, like the disappearance of water in the process described above, merely as an escape in various ways from the immediate scope of our observation. But so numerous, and in many cases obscure, are these ways of escapes, that their full extent and significance has been perceived only within quite recent years.

The belief that energy cannot be destroyed or diminished, and the converse belief that energy cannot be created or increased, — which latter belief is practically rejected by those who attempt to "invent perpetual motion," — constitute the doctrine of the Conservation of Energy. All the familiar devices for the advantageous application of mechanical power, such as the Lever, the Inclined Plane, and the Hydraulic Press, are simple examples of the truth of this doctrine, and their laws may be at once deduced from it.

What is now called the *Conservation of Energy* was formerly called the *Conservation of Force*, a phrase still sometimes used. The name "force" was thus applied to that which is now called energy. At the same time it

had the signification to which in these pages it has been confined, — a push or a pull. This ambiguity of the word was not always perceived or remembered by those who used the phrase "conservation of force," and the result was a confusion of thought which must have seriously retarded the acceptance of the truth that phrase was intended to set forth. Even at the present day it has not ceased to be a stumbling-block to some eminent thinkers, and it should not be left to trip the feet of the beginner.

APPENDIX.

THE definition of inertia given on page 8 of this pamphlet was suggested by the following passage, and another similar to it, in Stokes' little book, *On the Nature of Light:*[1] "To account for undulations in this medium, we must attach to it the two radical conceptions of inertia and elasticity. First, a finite time must be required to generate in a finite portion of it a finite velocity by the action of a finite force," etc.

A common definition of inertia is, the inability of matter to set itself in motion or to stop itself. If we define it in this way, simply as an *inability*, we are not at liberty to regard one body as having more or less inertia than another body, for evidently we cannot say that one body has more inability than another body. Now the use of the word inertia to denote a measurable property — a property of which one body may possess more or less than another body — is, as will presently be shown, by no means uncommon among physicists. The definition given in this pamphlet is perfectly consistent with this use, and at the same time it recognizes the inability which the other definition declares.

It has been suggested, that, if there were need of a special name for this inability, *inertness* would, perhaps, convey nearly the proper notion; but it seems hardly more necessary to give a special name to a body's inabil-

[1] Macmillan, 1884.

ity to change its own motion than to its inability to bend itself or to liquefy itself.

Precedent and the authority of distinguished writers must be chiefly effective in deciding the meaning of terms; and since there is sometimes sharp dispute as to what is the practice of good authorities in using the word inertia, certain quotations will not be out of place here.

In Art. 216, vol. i., of Thomson and Tait's *Natural Philosophy* occurs the passage: "Matter has an innate power of resisting external influences, so that every body, as far as it can, remains at rest, or moves uniformly in a straight line. This, the *inertia* of matter, is proportional to the quantity of matter in the body." It has been suggested that the second sentence of this passage should read thus: This, the inertia *of a body*, etc. This criticism seems to be a valid one, but there can be no doubt that it was the intention of the authors to regard inertia as a *quantitative* property of matter, a property admitting of measurement.

Maxwell, writing in *Nature*, vol. xx. p. 214, criticises the first sentence of this statement in the following words: "Is it a fact that 'matter' has any power, either innate or acquired, of resisting external influences? Does not every force which acts upon a body always produce exactly that change in the motion of the body by which its value, as a force, is reckoned? Is a cup of tea to be accused of having an innate power of resisting the sweetening influence of sugar, because it persistently refuses to turn sweet unless the sugar is actually put into it?"

This passage has made a strong impression upon some readers, and it is therefore necessary to call attention to the fact that these sentences of Maxwell are somewhat difficult to reconcile with other statements in his writings. In chapter iv. of his *Theory of Heat*, in defining

work, he says: "Work is done when resistance is overcome." On page 101 of his *Matter and Motion* we read: "Work is the act of producing a change of configuration in a system in opposition to a force which resists that change." There can be no doubt that he regarded the act of setting a body in motion as the performance of work; that is, the overcoming of a resistance. Hence his criticism of Thomson and Tait is puzzling. The fact seems to be that Maxwell, perceiving that the words of Thomson and Tait might be interpreted as ascribing to matter a power of resisting in such a way, as to remain absolutely undisturbed by a small force, was thus led to use hastily as a critic language inconsistent with that which he used as an author.

Maxwell seldom used the word inertia. In chapter iv. of his *Theory of Heat* he once uses *mass* in the same sense in which others use *inertia*, saying that "the sole unalterable property of matter is its *mass*." Elsewhere in the same chapter, however, he speaks of mass as being equivalent to "quantity of matter." If we were to put these two passages together, we should get the statement that the sole unalterable property of matter is its quantity. The absurdity of this result shows simply that Maxwell in this chapter uses mass in two senses. It does not appear, however, that he habitually used mass in any other sense than that of quantity of matter. On the other hand, speaking of what happens when a stream of water flowing in a tube is suddenly stopped, he says, in Art. 548, vol. ii., of his *Electricity and Magnetism*, "These effects of the inertia of the fluid in the tube depend," etc. Apparently, however, he in general purposely avoided the word inertia.

On p. 51, vol. i., of his *Theory of Sound*, Lord Rayleigh says, speaking of tuning-forks: "the substance of the prongs near the bend may be reduced, the effect of

which is to diminish the force of the spring, leaving the inertia practically unchanged; or the inertia may be increased [. . .] "by loading the ends of the prongs," etc. On page 76 we find: "If any diminution be made in the inertia of any of the parts of a system," etc., and "Conversely any increase in the inertia increases," etc. Other passages from the book might be given, showing the same use of the word inertia.

On page 42 of O. J. Lodge's *Elementary Mechanics*, edition of 1879, occurs the passage: "Since inertia then is a characteristic property of all matter, it will serve to *measure* the quantity of matter in any given mass, and is always used for this purpose in Dynamics." And on the same page we read: "They have all the same inertia, and therefore the same quantity of matter," etc. Curiously enough, the preceding page of the book defines inertia as "a certain characteristic negative property" of matter, "which is simply its incompetency to change its own state, whether of rest or motion, by itself." This definition of inertia seems hardly consistent with the vigorous quantitative function which he assigns to it in the sentences first quoted.

In Bottomley's *Dynamics* (London and Glasgow, 1877) we find on page 14 the following "Definition. — Two bodies are said to be of equal mass when they have equal inertia."

In Anthony and Brackett's *Elementary Physics*, Part I. p. 6, the following statement is made concerning matter; "its distinctive characteristic is its persistence in whatever state of rest or motion it may happen to have, and the resistance which it offers to any attempt to change that state. This property is called *inertia*. It must be carefully distinguished from inactivity." On page 34 of the same book is the supplementary statement, "Inertia is not of itself a force, but the property of a

body, enabling it to offer a resistance to a change of motion."

This discussion may well conclude with a passage pointed out to the author of this pamphlet by Professor B. O. Peirce, and which occurs on page 107 of Minchin's *Uniplanar Kinematics.*

"Newton says: 'The *vis insita*, or innate force of matter, is a power of resisting by which every body, as much as in it lies, endeavors to persevere in its present state, whether it be of rest or of moving uniformly forward in a right line.' And in his remarks on the definition, he says that 'This *vis insita* may by a most significant name be called *vis inertiæ.* But a body exerts this force only when another force impressed upon it endeavors to change its condition; and the exercise of this force may be considered both as resistance and impulse; it is resistance in so far as the body, for maintaining its present state, withstands the force impressed,' etc.

"This terminology has been wholly ignored by English writers, and, as a result, the fact that a body exerts a *kick* (if we may use the expression for clearness of illustration) against any agent which acts on it by direct contact or through a medium for the purpose either of deviating its motion from a rectilinear course or of accelerating its velocity, has been lost sight of. The student must carefully observe that the force of inertia of a moving particle is not a force acting *on* the particle, but one exerted *by* it on some agent direct or indirect — a kick against change of motion."

The author has attempted in the body of this pamphlet to find the most definite interpretation of the phrase *quantity of matter*, as commonly employed in

text-books of physics in comparing bodies which are different in kind. As a result of this investigation, it becomes plain that the meaning of the phrase, so used, is, and must be, somewhat indefinite and illogical, for the word *matter* conveys the notion of a group of several properties not necessarily proportional to each other in different bodies, while the *quantity* of this matter is considered determined by reference to only one, or at most two, of these properties.

It may well be questioned whether a phrase the meaning of which is necessarily so indefinite, which will not allow itself to be looked in the face, deserves to be called a scientific phrase; but it appears to meet a certain want, and is found in the works of eminent physicists. The word *mass*, which is commonly defined in text-books of physics as equivalent to *quantity of matter*, has, when so used, nearly the same indefiniteness as the phrase which it replaces.

But when the word mass, or its symbol m, is used in strict mathematical reasoning, it must have a perfectly definite signification. The purely mathematical meaning of mass, and the purely mathematical meaning of inertia, are brought together by Rankine (*Applied Mechanics*, p. 482) in the following sentence: "The Mass, or Inertia, of a body, is a quantity proportional to the unbalanced force which is required in order to produce a given definite change in the motion of the body in a given interval of time." This definition, which makes mass and inertia the same thing, and which apparently makes them simply a *number*, gives to these terms all the meaning that is needed, or that can be expressed, in mathematical formulas. Similarly, force may be defined, and has been defined, as *rate of change of momentum*. Such definitions are admirable for their strict purpose in mathematics, but the author of this pamphlet believes that

they are not entirely adequate for the explanation of these terms as used in physical text-books, for it appears to him that physicists generally associate with the words inertia, mass, and force certain ideas which cannot be expressed in numbers.